



Recording of Mathematical Calculations

Intent

At Haggonfields Primary, we follow the 'White Rose Hub' Mathematics' block planning scheme. This builds on skills sequentially, alongside regular recap and revisiting. Through regular recall learned material is embedded in the long-term memory. A vital aspect of revisiting is to become secure and fluent in the four mathematical calculations; addition, subtraction, multiplication and division.

The 'White Rose Hub' as part of their provision have a substantial Calculation Policy for its members to follow

Addition and Subtraction

<https://assets.whiteroseeducation.com/new-schemes/Addition%20and%20subtraction%20calculation%20policy%20July%202022%20v2.pdf>

Multiplication and Division

<https://assets.whiteroseeducation.com/new-schemes/Multiplication%20and%20Division%20calculation%20policy%20July%202022.pdf>

At Haggonfields this Calculation Policy is used as an integral part of planning. Therefore, pupils' fluency and rapid recall is developed thus enabling the confident learning of the Mathematics' Curriculum (DfE, 2013).

This document has been written to highlight the journey that each mathematical calculation takes. The guidance shows how pupils' learning is modelled and scaffolded by 'Concrete' materials and 'Pictorial' representation in order that 'Abstract' recording is deeply understood. It is intended that this secure deep understanding enables a true mastery of mathematics. Skills can be transferred between learning blocks, reasoning can be confidently explained and problems considered, tackled and solved effectively. Skills are also presented in context so that learning is useful. Pupils' resilience is boosted by having a deep understanding of skills as the learning allows multiple methods of calculation to be accessed so that answers can be checked independently and miscalculations used as learning opportunities.

The final section of this policy identifies the teaching of multiplication tables in recognition of the essential role it plays in the calculating of multiplication and division.

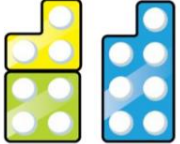
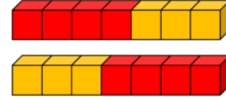
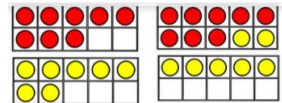
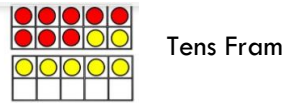


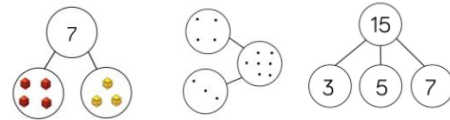
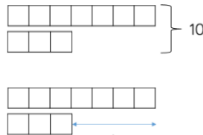

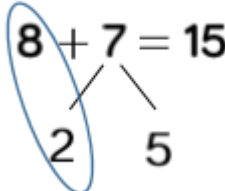
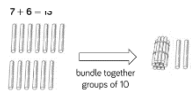
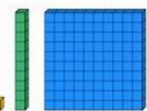
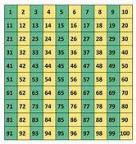
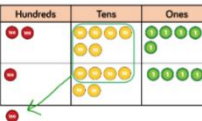
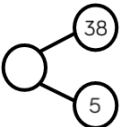
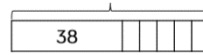
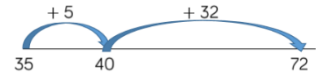
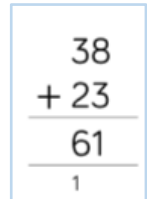
Aims

The aims of the calculation policy guidance at Haggonfields Primary is to provide:

- ★ An overview of the different models and images that can be used to support teaching.
- ★ Links between operations so that mathematics is taught with a mastery approach.

It is noted that concrete models and pictorial representation are not exclusive, a cumulative approach allows pupils to build upon skills and knowledge.

Addition

Skill	Concrete Model	Pictorial Representation	Abstract Recording
<p>Add two 1-digit numbers to 10</p> <p>Add 1 and 2-digit numbers to 20</p> <p>Add three 1-digit numbers</p>	<p>Numicon  Cubes </p> <p>Tens Frame  Tens Frame </p> <p>Bead Strings </p> <p>Number Track </p>	<p>Part-Whole Model </p> <p>Bar Model </p> <p>Labelled Number Line </p>	$7 = 4 + 3$ $7 = 3 + 4$ $8 + 7 = 15$ 
<p>Add 1-digit and 2-digit numbers to 100</p> <p>Add two 2-digit numbers</p> <p>Add numbers with up to 3 digits</p>	<p>Straws  bundle together groups of 10</p> <p>Dienes </p> <p>Hundred Square </p> <p>PV counters </p>	<p>Part-Whole Model </p> <p>Bar Model </p> <p>Labelled and Blank Number Lines </p>	$265 + 164 = 429$ 

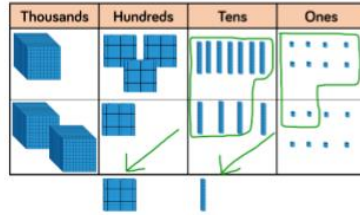
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Add numbers with up to 4 digits

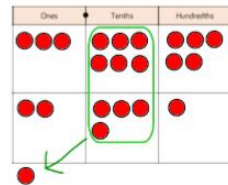
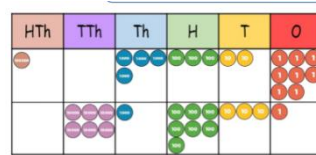
Add numbers with more than 4 digits

Add numbers with up to 3 decimal places

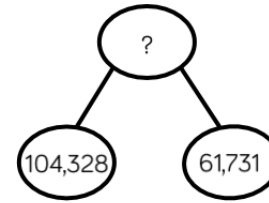
Dienes



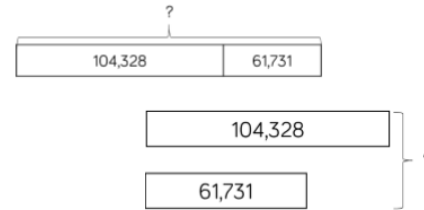
PV counters



Part-Whole Model



Bar Model



$$104,328 + 61,731 = 166,059$$

$$\begin{array}{r} 3.65 \\ + 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$

It is noted that concrete models and pictorial representation are not exclusive, a cumulative approach allows pupils to build upon skills and knowledge.

Subtraction

Skill	Concrete Model	Pictorial Representation	Abstract Recording
<p>Subtract two 1-digit numbers to 10</p> <p>Subtract 1 and 2- digit numbers to 20</p>	<p>Numicon</p> <p>Bead Strings</p> <p>Tens Frame</p> <p>First: Then: Now: <p>Number Track</p> </p>	<p>Part-Whole Model</p> <p>Bar Model</p> <p>Labelled Number Line</p>	$7 - 3 = 4$
<p>Subtract 1 and 2-digit numbers to 100</p> <p>Subtract numbers with up to 3 digits</p>	<p>Straws</p> <p>Hundred Square</p> <p>Dienes</p> <p>PV counters</p>	<p>Part-Whole Model</p> <p>Bar Model</p> <p>Labelled and Blank Number Lines</p> <p>$416 - 120 =$</p>	$65 - 28 = 37$

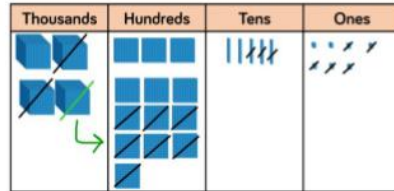
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Subtract numbers with up to 4 digits

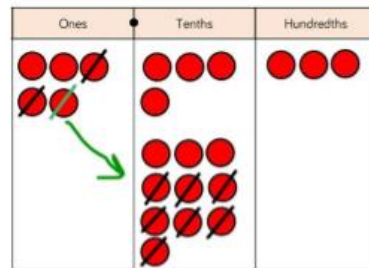
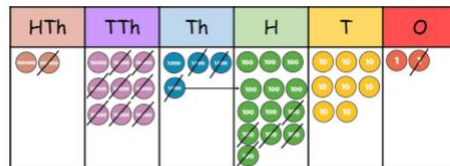
Subtract numbers with more than 4 digits

Subtract numbers with up to 3 decimal places

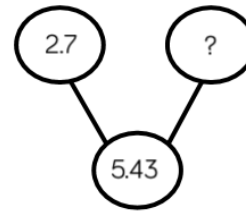
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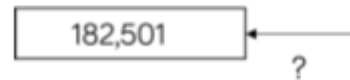
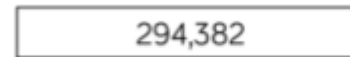
PV counters



Part-Whole Model



Bar Model


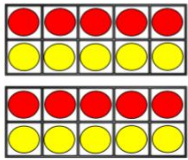
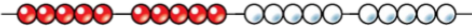
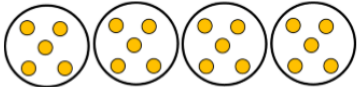
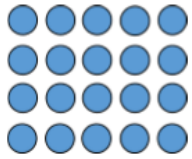
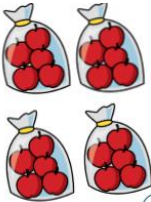
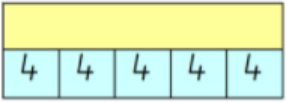
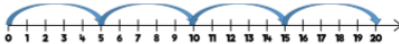


$$4,357 - 2,735 = 1,622$$

$$\begin{array}{r} 4 \ 1 \\ 5.43 \\ - 2.7 \\ \hline 2.73 \end{array}$$

It is noted that concrete models and pictorial representation are not exclusive, a cumulative approach allows pupils to build upon skills and knowledge.

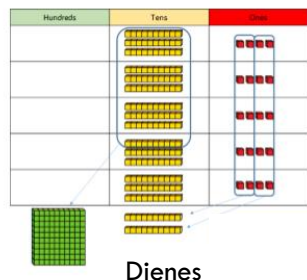
Multiplication

Skill	Concrete Model	Pictorial Representation	Abstract Recording
<p>Solve one step problems with multiplication</p> <p>One bag holds 5 apples. How many apples do 4 bags hold?</p> <p>Multiplication is represented as repeated addition in many ways.</p>	<p>Numicon</p>  <p>Tens Frame</p>  <p>Bead Strings</p>  <p>Counters</p>  <p>Arrays</p> 	<p>Drawing</p>  <p>Bar Model</p>  <p>Labelled Number Line</p> 	$5 + 5 + 5 + 5 = 20$ $4 \times 5 = 20$ $5 \times 4 = 20$

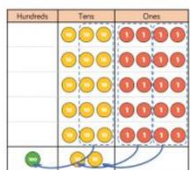
It is noted that concrete models and pictorial representation are not exclusive, a cumulative approach allows pupils to build upon skills and knowledge.

Multiply 2-digit by 1-digit numbers
 Multiply 3-digit by 1-digit numbers
 Multiply 4-digit by 1-digit numbers

Representation is used to support the understanding of these methods as times table knowledge should be firmly embedded.

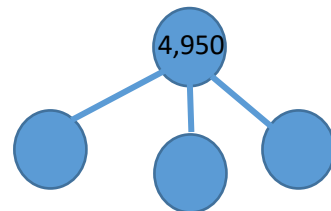


PV counters

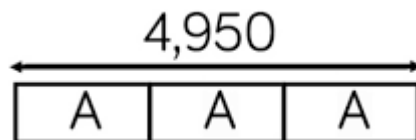


Cuisenaire Rods

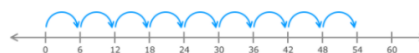
Part-Whole Model



Bar Model



Blank Number Lines



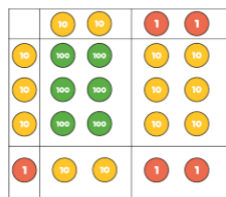
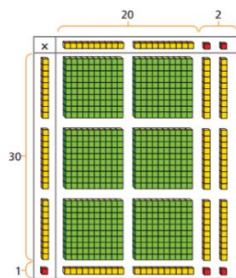
$$34 \times 5 = 170$$

	H	T	O	
		3	4	
x			5	
		2	0	(5 × 4)
+	1	5	0	(5 × 30)
	1	7	0	

	H	T	O
		3	4
x			5
	1	7	0
	1	2	

Multiply 2-digit by 2-digit numbers
 Multiply 3-digit by 2-digit numbers
 Multiply 4-digit by 2-digit numbers

Dienes



PV counters

When multiplying a multi-digit number by 2-digits use the area model to help understand the size of the numbers.




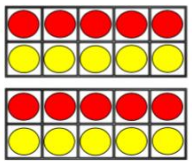

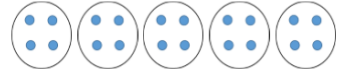
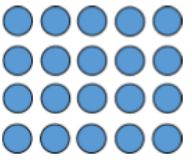


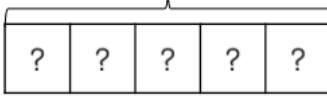
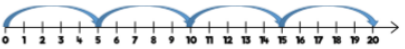
$$234 \times 32 = 7,488$$

x	200	30	4
30	6,000	900	120
2	400	60	8

	TTh	Th	H	T	O
		2	7	3	9
x				2	8
2	1	9	1	2	
2	5	3	7		
1	5	4	7	8	0
	7	6	6	9	2

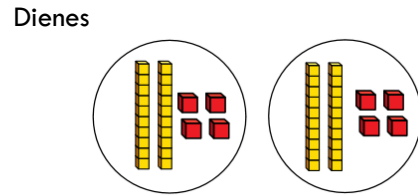
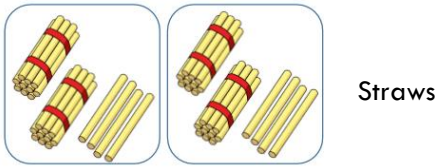
It is noted that concrete models and pictorial representation are not exclusive, a cumulative approach allows pupils to build upon skills and knowledge.

Division

Skill	Concrete Model	Pictorial Representation	Abstract Recording
<p>Solve one step problems with division (sharing)</p> <p>Solve one step problems with division (grouping)</p> <div style="border: 1px solid black; border-radius: 10px; padding: 5px; margin-top: 10px;"> <p>There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?</p> </div>	<p style="text-align: center;">Numicon</p>  <p style="text-align: center;">Tens Frame</p>  <p style="text-align: center;">Bead Strings</p>  <p style="text-align: center;">Counters</p>  <p style="text-align: center;">Arrays</p> 	<p style="text-align: center;">Drawing</p>   <p style="text-align: center;">Bar Model</p> <p style="text-align: center;">20</p>  <p style="text-align: center;">Labelled Number Line</p> 	<div style="border: 1px solid black; padding: 10px; margin: 10px;"> <p>Initially division is solved using concrete materials and pictorial representations. When confidence grows the division symbol is introduced.</p> </div> <div style="text-align: center; margin-top: 20px;"> $20 \div 5 = 4$ </div>

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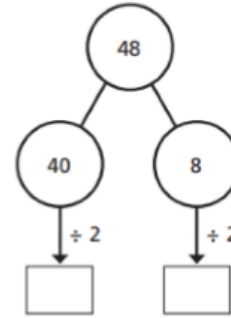
Divide 2-digit by 1-digit numbers
(no exchange)



PV counters

Tens	Ones
10 10	1 1 1 1
10 10	1 1 1 1

Part-Whole Model

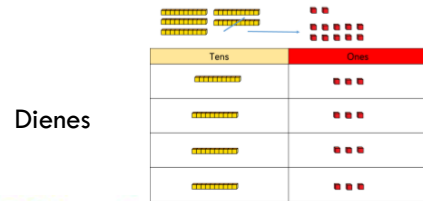
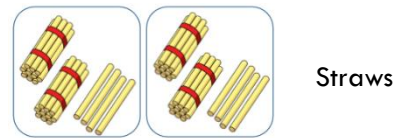


$$48 \div 2 = 24$$

Divide 2-digit by 1-digit numbers
(sharing no exchange)

Divide 2-digit by 1-digit numbers
(sharing with exchange)

Divide 2-digit by 1-digit numbers
(sharing with remainders)



PV counters

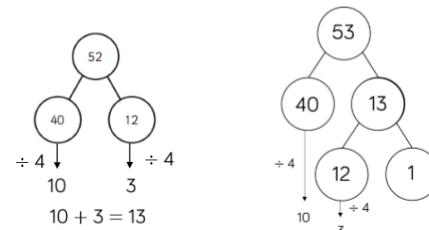
Tens	Ones
10	1 1 1 1
10	1 1 1 1
10	1 1 1 1
10	1 1 1 1

Bar Model



$$52 \div 4 = 13$$

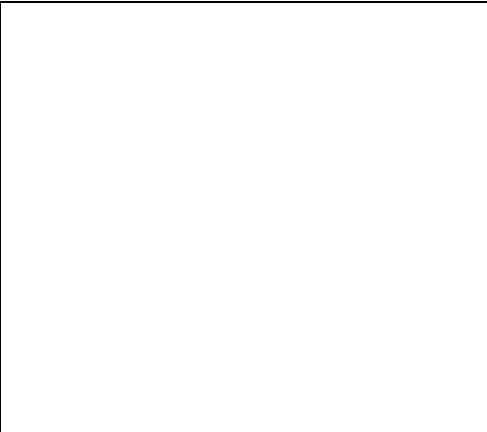
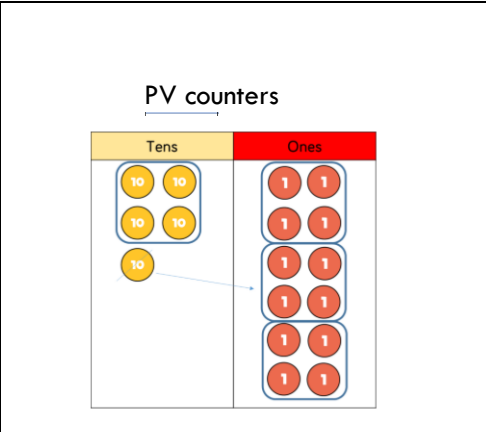
Part-Whole Model



$$53 \div 4 = 13 \text{ r}1$$

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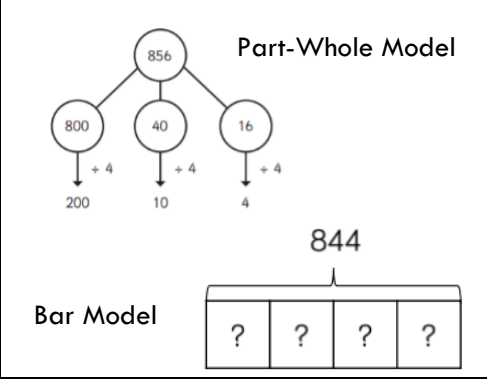
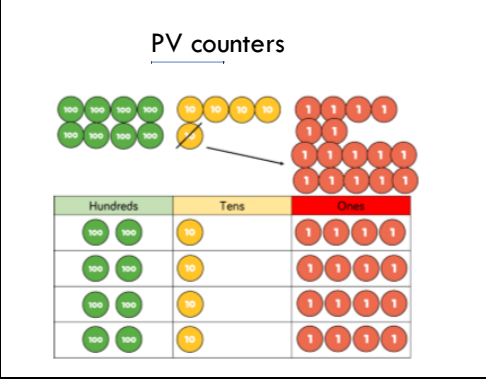
Divide 2-Digit by 1-Digit (grouping)



$$52 \div 4 = 13$$

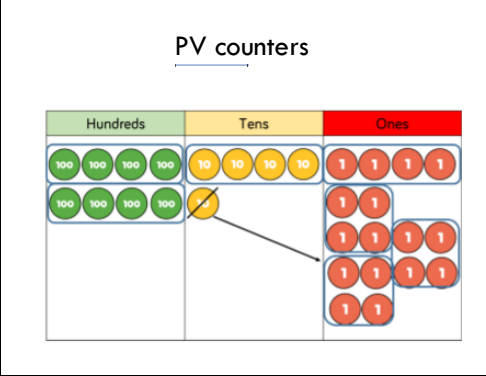


Divide 3-Digit by 1-Digit (sharing)



$$844 \div 4 = 211$$

Divide 3-Digit by 1-Digit (grouping)
Divide 4-Digit by 1-Digit (grouping)



$$856 \div 4 = 214$$


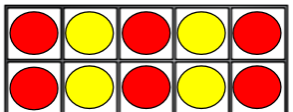
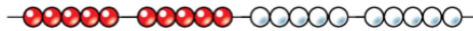

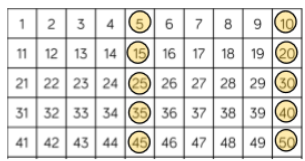

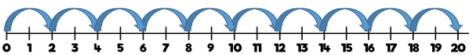


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<p>Divide Multi-Digit by 2-Digit (short division)</p>			<table border="1" style="margin-bottom: 10px;"> <tr><td></td><td></td><td>0</td><td>3</td><td>6</td></tr> <tr><td></td><td>12</td><td>4</td><td>4</td><td>7</td></tr> <tr><td></td><td></td><td></td><td>3</td><td>2</td></tr> </table> <div style="border: 1px solid black; border-radius: 10px; padding: 5px; display: inline-block;">$432 \div 12 = 36$</div> <table border="1" style="margin-bottom: 10px;"> <tr><td></td><td></td><td>0</td><td>4</td><td>8</td><td>9</td></tr> <tr><td></td><td>15</td><td>7</td><td>7</td><td>13</td><td>13</td></tr> <tr><td></td><td></td><td></td><td>3</td><td>3</td><td>5</td></tr> </table> <div style="border: 1px solid black; border-radius: 10px; padding: 5px; display: inline-block;">$7,335 \div 15 = 489$</div> <table border="1" style="width: 100%; text-align: center;"> <tr> <td>15</td><td>30</td><td>45</td><td>60</td><td>75</td><td>90</td><td>105</td><td>120</td><td>135</td><td>150</td> </tr> </table>			0	3	6		12	4	4	7				3	2			0	4	8	9		15	7	7	13	13				3	3	5	15	30	45	60	75	90	105	120	135	150																																									
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<p>Divide Multi-Digit by 2-Digit (long division)</p>			<div style="display: flex; justify-content: space-between; align-items: center;"> <div style="border: 1px solid black; border-radius: 10px; padding: 5px; display: inline-block;">$372 \div 15 = 24 \text{ r}12$</div> <table border="1" style="margin-left: 20px;"> <tr><td></td><td></td><td></td><td>2</td><td>4</td><td>r</td><td>1</td><td>2</td></tr> <tr><td>1</td><td>5</td><td>3</td><td>7</td><td>2</td><td></td><td></td><td></td></tr> <tr><td>-</td><td>3</td><td>0</td><td>0</td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td>7</td><td>2</td><td></td><td></td><td></td></tr> <tr><td>-</td><td></td><td>6</td><td>0</td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td>1</td><td>2</td><td></td><td></td><td></td></tr> </table> <div style="margin-left: 20px;"> <p>$1 \times 15 = 15$</p> <p>$2 \times 15 = 30$</p> <p>$3 \times 15 = 45$</p> <p>$4 \times 15 = 60$</p> <p>$5 \times 15 = 75$</p> <p>$10 \times 15 = 150$</p> </div> </div> <table border="1" style="margin-bottom: 10px;"> <tr><td></td><td></td><td></td><td>2</td><td>4</td><td>$\frac{4}{5}$</td></tr> <tr><td>1</td><td>5</td><td>3</td><td>7</td><td>2</td><td></td></tr> <tr><td>-</td><td>3</td><td>0</td><td>0</td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td>7</td><td>2</td><td></td></tr> <tr><td>-</td><td></td><td>6</td><td>0</td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td>1</td><td>2</td><td></td></tr> </table> <div style="border: 1px solid black; border-radius: 10px; padding: 5px; display: inline-block;">$372 \div 15 = 24 \frac{4}{5}$</div>				2	4	r	1	2	1	5	3	7	2				-	3	0	0								7	2				-		6	0								1	2							2	4	$\frac{4}{5}$	1	5	3	7	2		-	3	0	0						7	2		-		6	0						1	2	
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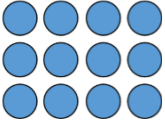
Multiplication Tables

Skill	Concrete Model	Visual Representation	Recall
<p>Recall and use the multiplication facts for the 2, 5 and 10 times' table.</p>	<p>Numicon </p> <p> Tens Frame</p> <p> Bead Strings</p> <p>Everyday objects</p> <p></p> <p> Number Grid</p>	<p></p> <p>Counting stick</p> <p></p> <p>Labelled Number Line</p>	<p>Regular counting in multiples both forwards and backwards.</p> <p>Supported by a number line or hundred square.</p> <p>Look for patterns in the ones as well as odds and evens</p>

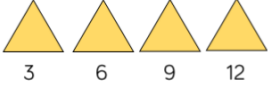
It is noted that concrete models and pictorial representation are not exclusive, a cumulative approach allows pupils to build upon skills and knowledge.

Recall and use the multiplication facts for the 3, 4 and 8 times' table.


Array



Shapes




Numicon



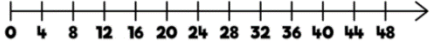
Number Grid

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

4	8	12	16	20
24	28	32	36	40
44	48	52	56	60



Counting stick



Labelled Number Lines

Regular counting in multiples both forwards and backwards.

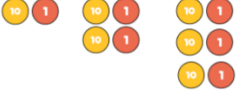
Supported by a number line or hundred square.

Look for patterns in the ones as well as odds and evens


Make links to previous tables (4 to 2 and 8 to 4)

Recall and use the multiplication facts for the 6, 9, 7, 11 and 12 times' table.

PV counters




Dienes



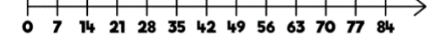
Number Grid

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

9	18	27	36	45
54	63	72	81	90



Counting stick



Labelled Number Lines

Regular counting in multiples both forwards and backwards.

Supported by a number line or hundred square.

Look for patterns in the ones as well as odds and evens

Make links to previous tables (9 and 6 to 3 and 12 to 4 and 6)

It is noted that concrete models and pictorial representation are not exclusive, a cumulative approach allows pupils to build upon skills and knowledge.